1.5.1 EXERCISES

To see all of the help resources associated with this section, click OSttS Chapter 1b.
In Exercises 1 - 10, use the pair of functions f and g to find the following values if they exist.

- $(f+g)(2)$
- $(f-g)(-1)$
- $(g-f)(1)$
- $(fg)\left(\frac{1}{2}\right)$
- $\left(\frac{f}{g}\right)(0)$
- $\left(\frac{g}{f}\right)(-2)$

For help with these exercises, click on one of the resources below:

- Adding and subtracting functions.
- Multiplying and dividing functions.
- Link to prerequisite algebra material (For help with complex fractions and radicals.)

1. $f(x) = 3x+1$ and $g(x) = 4-x$
2. $f(x) = x^2$ and $g(x) = -2x+1$
3. $f(x) = x^2-x$ and $g(x) = 12-x^2$
4. $f(x) = 2x^3$ and $g(x) = -x^2-2x-3$
5. $f(x) = \sqrt{x+3}$ and $g(x) = 2x-1$
6. $f(x) = \sqrt{4-x}$ and $g(x) = \sqrt{x+2}$
7. $f(x) = 2x$ and $g(x) = \dfrac{1}{2x+1}$
8. $f(x) = x^2$ and $g(x) = \dfrac{3}{2x-3}$
9. $f(x) = x^2$ and $g(x) = \dfrac{1}{x^2}$
10. $f(x) = x^2+1$ and $g(x) = \dfrac{1}{x^2+1}$

In Exercises 11 - 20, use the pair of functions f and g to find the domain of the indicated function then find and simplify an expression for it.

- $(f+g)(x)$
- $(f-g)(x)$
- $(fg)(x)$
- $\left(\frac{f}{g}\right)(x)$

For help with these exercises, click on one of the resources below:

- Adding and subtracting functions.
- Multiplying and dividing functions.
- Link to prerequisite algebra material (For help with complex fractions and radicals.)

11. $f(x) = 2x+1$ and $g(x) = x-2$
12. $f(x) = 1-4x$ and $g(x) = 2x-1$

13. $f(x) = x^2$ and $g(x) = 3x - 1$

14. $f(x) = x^2 - x$ and $g(x) = 7x$

15. $f(x) = x^2 - 4$ and $g(x) = 3x + 6$

16. $f(x) = -x^2 + x + 6$ and $g(x) = x^2 - 9$

17. $f(x) = \dfrac{x}{2}$ and $g(x) = \dfrac{2}{x}$

18. $f(x) = x - 1$ and $g(x) = \dfrac{1}{x-1}$

19. $f(x) = x$ and $g(x) = \sqrt{x+1}$

20. $f(x) = \sqrt{x-5}$ and $g(x) = f(x) = \sqrt{x-5}$

In Exercises 21 - 45, find and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for the given function. For help with these exercises, click on the resource below:

- Simplifying difference quotients
- Link to prerequisite algebra material (For help with complex fractions and radicals.)

21. $f(x) = 2x - 5$

22. $f(x) = -3x + 5$

23. $f(x) = 6$

24. $f(x) = 3x^2 - x$

25. $f(x) = -x^2 + 2x - 1$

26. $f(x) = 4x^2$

27. $f(x) = x - x^2$

28. $f(x) = x^3 + 1$

29. $f(x) = mx + b$ where $m \neq 0$

30. $f(x) = ax^2 + bx + c$ where $a \neq 0$

31. $f(x) = \dfrac{2}{x}$

32. $f(x) = \dfrac{3}{1-x}$

33. $f(x) = \dfrac{1}{x^2}$

34. $f(x) = \dfrac{2}{x+5}$

35. $f(x) = \dfrac{1}{4x-3}$

36. $f(x) = \dfrac{3x}{x+1}$

37. $f(x) = \dfrac{x}{x-9}$

38. $f(x) = \dfrac{x^2}{2x+1}$

39. $f(x) = \sqrt{x-9}$

40. $f(x) = \sqrt{2x+1}$

41. $f(x) = \sqrt{-4x+5}$

42. $f(x) = \sqrt{4-x}$

43. $f(x) = \sqrt{ax+b}$, where $a \neq 0$.

44. $f(x) = x\sqrt{x}$

45. $f(x) = \sqrt[3]{x}$. **HINT:** $(a-b)\left(a^2 + ab + b^2\right) = a^3 - b^3$

In Exercises 46 - 50, $C(x)$ denotes the cost to produce x items and $p(x)$ denotes the price-demand function in the given economic scenario. In each Exercise, do the following:

- Find and interpret $C(0)$.
- Find and interpret $\overline{C}(10)$.
- Find and interpret $p(5)$
- Find and simplify $R(x)$.
- Find and simplify $P(x)$.
- Solve $P(x) = 0$ and interpret.

46. The cost, in dollars, to produce x "I'd rather be a Sasquatch" T-Shirts is $C(x) = 2x + 26$, $x \geq 0$ and the price-demand function, in dollars per shirt, is $p(x) = 30 - 2x$, $0 \leq x \leq 15$.
47. The cost, in dollars, to produce x bottles of 100% All-Natural Certified Free-Trade Organic Sasquatch Tonic is $C(x) = 10x + 100$, $x \geq 0$ and the price-demand function, in dollars per bottle, is $p(x) = 35 - x$, $0 \leq x \leq 35$.
48. The cost, in cents, to produce x cups of Mountain Thunder Lemonade at Junior's Lemonade Stand is $C(x) = 18x + 240$, $x \geq 0$ and the price-demand function, in cents per cup, is $p(x) = 90 - 3x$, $0 \leq x \leq 30$.
49. The daily cost, in dollars, to produce x Sasquatch Berry Pies $C(x) = 3x + 36$, $x \geq 0$ and the price-demand function, in dollars per pie, is $p(x) = 12 - 0.5x$, $0 \leq x \leq 24$.
50. The monthly cost, in hundreds of dollars, to produce x custom built electric scooters is $C(x) = 20x + 1000$, $x \geq 0$ and the price-demand function, in hundreds of dollars per scooter, is $p(x) = 140 - 2x$, $0 \leq x \leq 70$.

In Exercises 51 - 62, let f be the function defined by

$$f = \{(-3,4), (-2,2), (-1,0), (0,1), (1,3), (2,4), (3,-1)\}$$

and let g be the function defined by

$$g = \{(-3,-2), (-2,0), (-1,-4), (0,0), (1,-3), (2,1), (3,2)\}$$

Compute the indicated value if it exists.

51. $(f+g)(-3)$
52. $(f-g)(2)$
53. $(fg)(-1)$
54. $(g+f)(1)$
55. $(g-f)(3)$
56. $(gf)(-3)$
57. $\left(\frac{f}{g}\right)(-2)$
58. $\left(\frac{f}{g}\right)(-1)$
59. $\left(\frac{f}{g}\right)(2)$
60. $\left(\frac{g}{f}\right)(-1)$
61. $\left(\frac{g}{f}\right)(3)$
62. $\left(\frac{g}{f}\right)(-3)$

Checkpoint Quiz 1.5

1. Let $f(x) = 2x + 6$ and $g(x) = x^2 - 9$.

   Find the domain of the following functions and simplify their expressions.

   (a) $(g - f)(x)$

   (b) $\left(\dfrac{f}{g}\right)(x)$

2. Let $f(x) = -x^2 + 2x - 3$. Find and simplify the difference quotient, $\dfrac{f(x+h) - f(x)}{h}$.

For worked out solutions to this quiz, click the link below:

- Quiz Solution

1.5.2 Answers

1. For $f(x) = 3x + 1$ and $g(x) = 4 - x$
   - $(f + g)(2) = 9$
   - $(f - g)(-1) = -7$
   - $(g - f)(1) = -1$
   - $(fg)\left(\frac{1}{2}\right) = \frac{35}{4}$
   - $\left(\frac{f}{g}\right)(0) = \frac{1}{4}$
   - $\left(\frac{g}{f}\right)(-2) = -\frac{6}{5}$

2. For $f(x) = x^2$ and $g(x) = -2x + 1$
   - $(f + g)(2) = 1$
   - $(f - g)(-1) = -2$
   - $(g - f)(1) = -2$
   - $(fg)\left(\frac{1}{2}\right) = 0$
   - $\left(\frac{f}{g}\right)(0) = 0$
   - $\left(\frac{g}{f}\right)(-2) = \frac{5}{4}$

3. For $f(x) = x^2 - x$ and $g(x) = 12 - x^2$
   - $(f + g)(2) = 10$
   - $(f - g)(-1) = -9$
   - $(g - f)(1) = 11$
   - $(fg)\left(\frac{1}{2}\right) = -\frac{47}{16}$
   - $\left(\frac{f}{g}\right)(0) = 0$
   - $\left(\frac{g}{f}\right)(-2) = \frac{4}{3}$

4. For $f(x) = 2x^3$ and $g(x) = -x^2 - 2x - 3$
   - $(f + g)(2) = 5$
   - $(f - g)(-1) = 0$
   - $(g - f)(1) = -8$
   - $(fg)\left(\frac{1}{2}\right) = -\frac{17}{16}$
   - $\left(\frac{f}{g}\right)(0) = 0$
   - $\left(\frac{g}{f}\right)(-2) = \frac{3}{16}$

5. For $f(x) = \sqrt{x + 3}$ and $g(x) = 2x - 1$
   - $(f + g)(2) = 3 + \sqrt{5}$
   - $(f - g)(-1) = 3 + \sqrt{2}$
   - $(g - f)(1) = -1$
   - $(fg)\left(\frac{1}{2}\right) = 0$
   - $\left(\frac{f}{g}\right)(0) = -\sqrt{3}$
   - $\left(\frac{g}{f}\right)(-2) = -5$

6. For $f(x) = \sqrt{4 - x}$ and $g(x) = \sqrt{x + 2}$
   - $(f + g)(2) = 2 + \sqrt{2}$
   - $(f - g)(-1) = -1 + \sqrt{5}$
   - $(g - f)(1) = 0$
   - $(fg)\left(\frac{1}{2}\right) = \frac{\sqrt{35}}{2}$
   - $\left(\frac{f}{g}\right)(0) = \sqrt{2}$
   - $\left(\frac{g}{f}\right)(-2) = 0$

7. For $f(x) = 2x$ and $g(x) = \frac{1}{2x+1}$
   - $(f+g)(2) = \frac{21}{5}$
   - $(f-g)(-1) = -1$
   - $(g-f)(1) = -\frac{5}{3}$
   - $(fg)\left(\frac{1}{2}\right) = \frac{1}{2}$
   - $\left(\frac{f}{g}\right)(0) = 0$
   - $\left(\frac{g}{f}\right)(-2) = \frac{1}{12}$
8. For $f(x) = x^2$ and $g(x) = \frac{3}{2x-3}$
   - $(f+g)(2) = 7$
   - $(f-g)(-1) = \frac{8}{5}$
   - $(g-f)(1) = -4$
   - $(fg)\left(\frac{1}{2}\right) = -\frac{3}{8}$
   - $\left(\frac{f}{g}\right)(0) = 0$
   - $\left(\frac{g}{f}\right)(-2) = -\frac{3}{28}$
9. For $f(x) = x^2$ and $g(x) = \frac{1}{x^2}$
   - $(f+g)(2) = \frac{17}{4}$
   - $(f-g)(-1) = 0$
   - $(g-f)(1) = 0$
   - $(fg)\left(\frac{1}{2}\right) = 1$
   - $\left(\frac{f}{g}\right)(0)$ is undefined.
   - $\left(\frac{g}{f}\right)(-2) = \frac{1}{16}$
10. For $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x^2+1}$
    - $(f+g)(2) = \frac{26}{5}$
    - $(f-g)(-1) = \frac{3}{2}$
    - $(g-f)(1) = -\frac{3}{2}$
    - $(fg)\left(\frac{1}{2}\right) = 1$
    - $\left(\frac{f}{g}\right)(0) = 1$
    - $\left(\frac{g}{f}\right)(-2) = \frac{1}{25}$
11. For $f(x) = 2x + 1$ and $g(x) = x - 2$
    - $(f+g)(x) = 3x - 1$
      Domain: $(-\infty, \infty)$
    - $(f-g)(x) = x + 3$
      Domain: $(-\infty, \infty)$
    - $(fg)(x) = 2x^2 - 3x - 2$
      Domain: $(-\infty, \infty)$
    - $\left(\frac{f}{g}\right)(x) = \frac{2x+1}{x-2}$
      Domain: $(-\infty, 2) \cup (2, \infty)$
12. For $f(x) = 1 - 4x$ and $g(x) = 2x - 1$
    - $(f+g)(x) = -2x$
      Domain: $(-\infty, \infty)$
    - $(f-g)(x) = 2 - 6x$
      Domain: $(-\infty, \infty)$
    - $(fg)(x) = -8x^2 + 6x - 1$
      Domain: $(-\infty, \infty)$
    - $\left(\frac{f}{g}\right)(x) = \frac{1-4x}{2x-1}$
      Domain: $\left(-\infty, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

13. For $f(x) = x^2$ and $g(x) = 3x - 1$

- $(f+g)(x) = x^2 + 3x - 1$
  Domain: $(-\infty, \infty)$
- $(f-g)(x) = x^2 - 3x + 1$
  Domain: $(-\infty, \infty)$
- $(fg)(x) = 3x^3 - x^2$
  Domain: $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x^2}{3x-1}$
  Domain: $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, \infty\right)$

14. For $f(x) = x^2 - x$ and $g(x) = 7x$

- $(f+g)(x) = x^2 + 6x$
  Domain: $(-\infty, \infty)$
- $(f-g)(x) = x^2 - 8x$
  Domain: $(-\infty, \infty)$
- $(fg)(x) = 7x^3 - 7x^2$
  Domain: $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x-1}{7}$
  Domain: $(-\infty, 0) \cup (0, \infty)$

15. For $f(x) = x^2 - 4$ and $g(x) = 3x + 6$

- $(f+g)(x) = x^2 + 3x + 2$
  Domain: $(-\infty, \infty)$
- $(f-g)(x) = x^2 - 3x - 10$
  Domain: $(-\infty, \infty)$
- $(fg)(x) = 3x^3 + 6x^2 - 12x - 24$
  Domain: $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x-2}{3}$
  Domain: $(-\infty, -2) \cup (-2, \infty)$

16. For $f(x) = -x^2 + x + 6$ and $g(x) = x^2 - 9$

- $(f+g)(x) = x - 3$
  Domain: $(-\infty, \infty)$
- $(f-g)(x) = -2x^2 + x + 15$
  Domain: $(-\infty, \infty)$
- $(fg)(x) = -x^4 + x^3 + 15x^2 - 9x - 54$
  Domain: $(-\infty, \infty)$
- $\left(\frac{f}{g}\right)(x) = -\frac{x+2}{x+3}$
  Domain: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

17. For $f(x) = \frac{x}{2}$ and $g(x) = \frac{2}{x}$

- $(f+g)(x) = \frac{x^2+4}{2x}$
  Domain: $(-\infty, 0) \cup (0, \infty)$
- $(f-g)(x) = \frac{x^2-4}{2x}$
  Domain: $(-\infty, 0) \cup (0, \infty)$
- $(fg)(x) = 1$
  Domain: $(-\infty, 0) \cup (0, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x^2}{4}$
  Domain: $(-\infty, 0) \cup (0, \infty)$

18. For $f(x) = x - 1$ and $g(x) = \frac{1}{x-1}$

- $(f+g)(x) = \frac{x^2-2x+2}{x-1}$
Domain: $(-\infty, 1) \cup (1, \infty)$
- $(f-g)(x) = \frac{x^2-2x}{x-1}$
Domain: $(-\infty, 1) \cup (1, \infty)$
- $(fg)(x) = 1$
Domain: $(-\infty, 1) \cup (1, \infty)$
- $\left(\frac{f}{g}\right)(x) = x^2 - 2x + 1$
Domain: $(-\infty, 1) \cup (1, \infty)$

19. For $f(x) = x$ and $g(x) = \sqrt{x+1}$

- $(f+g)(x) = x + \sqrt{x+1}$
Domain: $[-1, \infty)$
- $(f-g)(x) = x - \sqrt{x+1}$
Domain: $[-1, \infty)$
- $(fg)(x) = x\sqrt{x+1}$
Domain: $[-1, \infty)$
- $\left(\frac{f}{g}\right)(x) = \frac{x}{\sqrt{x+1}}$
Domain: $(-1, \infty)$

20. For $f(x) = \sqrt{x-5}$ and $g(x) = f(x) = \sqrt{x-5}$

- $(f+g)(x) = 2\sqrt{x-5}$
Domain: $[5, \infty)$
- $(f-g)(x) = 0$
Domain: $[5, \infty)$
- $(fg)(x) = x - 5$
Domain: $[5, \infty)$
- $\left(\frac{f}{g}\right)(x) = 1$
Domain: $(5, \infty)$

21. 2
22. -3
23. 0
24. $6x + 3h - 1$
25. $-2x - h + 2$
26. $8x + 4h$
27. $-2x - h + 1$
28. $3x^2 + 3xh + h^2$
29. m
30. $2ax + ah + b$
31. $\dfrac{-2}{x(x+h)}$
32. $\dfrac{3}{(1-x-h)(1-x)}$
33. $\dfrac{-(2x+h)}{x^2(x+h)^2}$
34. $\dfrac{-2}{(x+5)(x+h+5)}$
35. $\dfrac{-4}{(4x-3)(4x+4h-3)}$
36. $\dfrac{3}{(x+1)(x+h+1)}$

37. $\dfrac{-9}{(x-9)(x+h-9)}$

38. $\dfrac{2x^2+2xh+2x+h}{(2x+1)(2x+2h+1)}$

39. $\dfrac{1}{\sqrt{x+h-9}+\sqrt{x-9}}$

40. $\dfrac{2}{\sqrt{2x+2h+1}+\sqrt{2x+1}}$

41. $\dfrac{-4}{\sqrt{-4x-4h+5}+\sqrt{-4x+5}}$

42. $\dfrac{-1}{\sqrt{4-x-h}+\sqrt{4-x}}$

43. $\dfrac{a}{\sqrt{ax+ah+b}+\sqrt{ax+b}}$

44. $\dfrac{3x^2+3xh+h^2}{(x+h)^{3/2}+x^{3/2}}$

45. $\dfrac{1}{(x+h)^{2/3}+(x+h)^{1/3}x^{1/3}+x^{2/3}}$

46.
- $C(0) = 26$, so the fixed costs are \$26.
- $\overline{C}(10) = 4.6$, so when 10 shirts are produced, the cost per shirt is \$4.60.
- $p(5) = 20$, so to sell 5 shirts, set the price at \$20 per shirt.
- $R(x) = -2x^2 + 30x$, $0 \leq x \leq 15$
- $P(x) = -2x^2 + 28x - 26$, $0 \leq x \leq 15$
- $P(x) = 0$ when $x = 1$ and $x = 13$. These are the 'break even' points, so selling 1 shirt or 13 shirts will guarantee the revenue earned exactly recoups the cost of production.

47.
- $C(0) = 100$, so the fixed costs are \$100.
- $\overline{C}(10) = 20$, so when 10 bottles of tonic are produced, the cost per bottle is \$20.
- $p(5) = 30$, so to sell 5 bottles of tonic, set the price at \$30 per bottle.
- $R(x) = -x^2 + 35x$, $0 \leq x \leq 35$
- $P(x) = -x^2 + 25x - 100$, $0 \leq x \leq 35$
- $P(x) = 0$ when $x = 5$ and $x = 20$. These are the 'break even' points, so selling 5 bottles of tonic or 20 bottles of tonic will guarantee the revenue earned exactly recoups the cost of production.

48.
- $C(0) = 240$, so the fixed costs are 240¢ or \$2.40.
- $\overline{C}(10) = 42$, so when 10 cups of lemonade are made, the cost per cup is 42¢.
- $p(5) = 75$, so to sell 5 cups of lemonade, set the price at 75¢ per cup.
- $R(x) = -3x^2 + 90x$, $0 \leq x \leq 30$
- $P(x) = -3x^2 + 72x - 240$, $0 \leq x \leq 30$
- $P(x) = 0$ when $x = 4$ and $x = 20$. These are the 'break even' points, so selling 4 cups of lemonade or 20 cups of lemonade will guarantee the revenue earned exactly recoups the cost of production.

49.
- $C(0) = 36$, so the daily fixed costs are \$36.
- $\overline{C}(10) = 6.6$, so when 10 pies are made, the cost per pie is \$6.60.
- $p(5) = 9.5$, so to sell 5 pies a day, set the price at \$9.50 per pie.
- $R(x) = -0.5x^2 + 12x$, $0 \leq x \leq 24$
- $P(x) = -0.5x^2 + 9x - 36$, $0 \leq x \leq 24$
- $P(x) = 0$ when $x = 6$ and $x = 12$. These are the 'break even' points, so selling 6 pies or 12 pies a day will guarantee the revenue earned exactly recoups the cost of production.

50.
- $C(0) = 1000$, so the monthly fixed costs are 1000 *hundred* dollars, or \$100,000.
- $\overline{C}(10) = 120$, so when 10 scooters are made, the cost per scooter is 120 hundred dollars, or \$12,000.
- $p(5) = 130$, so to sell 5 scooters a month, set the price at 130 hundred dollars, or \$13,000 per scooter.
- $R(x) = -2x^2 + 140x$, $0 \leq x \leq 70$
- $P(x) = -2x^2 + 120x - 1000$, $0 \leq x \leq 70$
- $P(x) = 0$ when $x = 10$ and $x = 50$. These are the 'break even' points, so selling 10 scooters or 50 scooters a month will guarantee the revenue earned exactly recoups the cost of production.

51. $(f+g)(-3) = 2$

52. $(f-g)(2) = 3$

53. $(fg)(-1) = 0$

54. $(g+f)(1) = 0$

55. $(g-f)(3) = 3$

56. $(gf)(-3) = -8$

57. $\left(\frac{f}{g}\right)(-2)$ does not exist

58. $\left(\frac{f}{g}\right)(-1) = 0$

59. $\left(\frac{f}{g}\right)(2) = 4$

60. $\left(\frac{g}{f}\right)(-1)$ does not exist

61. $\left(\frac{g}{f}\right)(3) = -2$

62. $\left(\frac{g}{f}\right)(-3) = -\frac{1}{2}$